

TTK-10-57  
January 18, 2011

## On the Status of $V_{td}$ , $V_{ts}$ and $V_{tb}$

JÜRGEN ROHRWILD

*Institut für theoretische Teilchenphysik und Kosmologie  
RWTH Aachen, D-52074 Aachen, GERMANY*

The CKM matrix elements  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$ , while notoriously hard to measure directly in experiment, can be determined to great precision within the Standard Model (SM) with a minimal set of observables due to unitarity of  $V_{\text{CKM}}$ . We discuss the potential modifications of these elements in extensions with non-unitary quark mixing matrices.

Proceedings of CKM2010, the 6th International Workshop on the  
CKM Unitarity Triangle

PRESENTED AT

University of Warwick, UK, 6-10 September 2010

# 1 Introduction

The precise determination of CKM matrix elements—be it their absolute value or the relative phases—and the verification of CP violation via the CKM mechanism [1] were milestones of particle physics in the last decade(s). Leptonic and semi-leptonic meson decays as well as superallowed  $\beta$  decay have provided an excellent idea about the absolute values of the element of the first two rows of the CKM matrix.

Unfortunately, direct measurements of the ‘*top*’-row elements cannot be carried out in similar fashion as the  $t$  width is too large for it to hadronize. Hence, direct information on these elements is fairly rare. On the other hand, a large amount of loop-dominated observables are very sensitive to the  $V_{tx}$  elements, e.g.,  $B_d$  mixing,  $B_s$  mixing, or  $b \rightarrow s\gamma$ , and thus provide indirect information on these elements. We use “indirect” to emphasise that one usually makes use of some Standard Model (SM) properties (e.g. CKM unitarity) to extract the desired information.

This also raises the question to what extent a non-standard CKM matrix arising in the low energy limit from some beyond the SM (BSM) scenario can differ from the SM form, if one takes into account the large amount of precision studies dedicated to the determination of the CKM matrix. In the following I will discuss the high precision provided by the unitarity condition in the SM and some BSM scenarios which lead to notable deviations from the SM values for the  $V_{tx}$  elements.

## 2 Experiment and Standard Model

Before examining the SM predictions for the *top* row matrix elements, let us first look at the few direct experimental constraints. There are, in principle, two ways to directly constrain the matrix element  $V_{tb}^*$ :  $t\bar{t}$  and single top production.

- $t\bar{t}$  production allows for a measurement of the ratio  $\mathcal{R}_b$  which is defined as

$$\mathcal{R}_b = \frac{\mathcal{B}[t \rightarrow Wb]}{\mathcal{B}[t \rightarrow Wq]} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} . \quad (1)$$

DØ determined  $\mathcal{R}_b$  to be  $0.97_{-0.08}^{+0.09}$  [2]. This immediately translates to a value for  $V_{tb} = 0.97_{-0.08}^{+0.09}$  in the SM. However, the model-independent conclusion is merely  $V_{tb} \gg V_{td}, V_{ts}$  as one cannot use unitarity.

- The single top production cross section is directly proportional to  $|V_{tb}|^2$ . The combined CDF- DØ result [3] (assuming a top quark mass of 170 GeV) leads to  $|V_{tb}| = 0.88 \pm 0.07 \pm 0.07$  (exp + theo) (see [4] for a recent CDF update and Wolfgang Wagner’s talk [5] for plots and details). Note that LHC can reduce the experimental uncertainty by almost a factor of 2 [6, 7, 8].

---

\*For specific BSM scenarios other options may exist.

From the SM point of view the issue of  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$  is of course already settled. The easiest way to determine not only the absolute values but also relative phases is the use of CKM unitarity which reduces the number of independent parameters to 4. In fact one can even avoid using any 'loop-related' observables by only making use of the absolute values of  $|V_{ud}|$ ,  $|V_{us}|$ ,  $|V_{ub}|$ ,  $|V_{cd}|$ ,  $|V_{cb}|$  and  $|V_{cs}|$  from tree-level decays and by supplying a single phase via the CKM angle  $\gamma$  from e.g.  $B \rightarrow D^{(*)}K$  which is tree-level dominated. The parameters of the CKM matrix can then be fitted; the UTfit group e.g. obtains [9]:

$$V_{CKM} = \begin{pmatrix} 0.9426 \pm 0.00015 & 0.22535 \pm 0.00065 & 0.00376 \pm 0.0002 \cdot e^{i(-73.8 \pm 9.4)^\circ} \\ -0.2252 \pm 0.00065 \cdot e^{i(-0.03656 \pm 0.0028)^\circ} & 0.97345 \pm 0.00015 & 0.04083 \pm 0.00045 \\ 0.00896 \pm 0.0006 \ \& \ 0.01081 \pm 0.0006 \cdot e^{i(-22.9 \pm 1.4)^\circ} & -0.03979 \pm 0.00052 \cdot e^{i(-1.163 \pm 0.084)^\circ} & 0.99916 \pm 1.8 \times 10^{-5} \end{pmatrix}$$

With exception of some ambiguity in  $V_{td}$  the values of  $V_{ts}$  and  $V_{tb}$  are already determined with astounding accuracy. However, one can do even better by including the plethora of precision flavour measurements in the analysis. This provides a powerful consistency check for unitarity and results in the famous unitarity triangle plots, see e.g. [10].

### 3 Beyond Unitarity

As we have seen in the previous section unitarity of the CKM matrix already pins down the magnitude of the  $V_{tx}$  elements to great accuracy even if conservative input data is used. Hence, a deviation from the SM values basically requires a breaking of the unitary condition. The most general parameterisation of the CKM matrix then requires 13 independent parameters, see e.g. [11]. Of course, it is not meaningful to ask for a general determination of this parameters as non-unitarity *must* be induced by some physics beyond the SM, whose effects would have to be taken into account.

Most scenarios generate the deviation of the CKM matrix by enlarging the fermion sector. Thus the SM CKM matrix is a  $3 \times 3$  subblock embedded in a larger fermion mixing matrix. While there are numerous BSM scenarios, this talk will focus only on two in a sense minimal models and briefly discuss a third, more involved extension.

#### 3.1 Additional Fermions

The simplest way to break unitarity of the SM CKM matrix is to either introduce one heavy vector-like quark or a complete additional (SM-like) fermion generation. In both cases the experimental data must, strictly speaking, be reinterpreted as theory input cannot make use of any identity like

$$V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{ud}V_{ub}^* = 0 \ .$$

Furthermore, the possibility of modifications to other well-known features of the SM flavour structure cannot be ruled out a priori.

**Vector-like quarks** generically make an appearance in various extensions of the Standard model like Randall-Sundrum scenarios or  $E_6$  GUTs. The minimal formulation just extends the fermion sector by a single heavy top or bottom like quark, see e.g. [12, 13]. Restricting to the case of a charge  $+2/3e$  vector quark  $T$  one finds the following features:

- The equivalent of the CKM matrix is now  $4 \times 3$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{pmatrix}. \quad (2)$$

- As the Yukawa matrix is no longer diagonal in the mass eigenbasis the Higgs interaction is flavour changing.
- The  $Z$  can induce flavour changing neutral currents (FCNCs). Their strength is proportional to the unitarity violation [13].
- The heavy  $T$  will preferably mix with the top quark; thus reproducing the unitary relations of first and second row within experimental accuracy.

The entries of the matrix in Eq. (2) and value of the mass of the  $T$  are subject to a large amount of experimental constraints coming from: direct measurement of CKM elements, meson mixing ( $\varepsilon_K, \varepsilon'/\varepsilon_K$ , mass differences in the  $B_{d/s}$  systems, ...), various rare kaon and  $B$  decays as well as bounds from the electroweak sector such as  $S$  and  $T$  parameter and the ratio  $R_b$  [14]. The two parameter sets below (taken from [13]) illustrate the size of the modification the CKM matrix (only absolute values of the elements are shown) can still experience:

$m_T = 450 \text{ GeV}$

$$|U_D| = \begin{pmatrix} 0.974179 & 0.225657 & 0.004031 \\ 0.225619 & 0.972525 & 0.041766 \\ 0.008330 & 0.047219 & 0.966377 \\ 0.001136 & 0.032304 & 0.253683 \end{pmatrix}$$

$m_T = 300 \text{ GeV}$

$$|U_D| = \begin{pmatrix} 0.974195 & 0.225663 & 0.004137 \\ 0.2254882 & 0.972938 & 0.041548 \\ 0.009721 & 0.042034 & 0.945531 \\ 0.002889 & 0.026471 & 0.322842 \end{pmatrix}$$

**One additional fermion generation:** Introducing a SM-like generation (adding 7 new parameters in the quark sector) is conceptionally even simpler than the vectorlike quark model. However, the absence of FCNCs and gauge anomalies comes with a massive fourth neutrino<sup>†</sup>. Both CKM and PMNS matrices have to be promoted to

---

<sup>†</sup>The impact of the large mass and the implications for the PMNS matrix are discussed e.g. in [15].

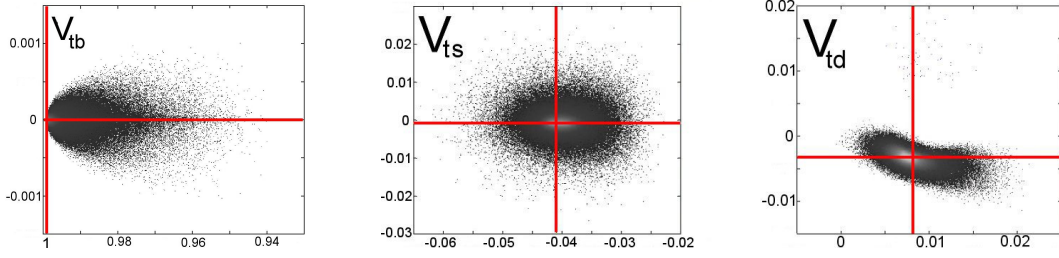


Figure 1: Scatterplots for  $V_{tb}$ ,  $V_{ts}$  and  $V_{td}$  (standard representation) in the complex plane for the 4G scenario [18]. The crossed lines denote the SM values.

$4 \times 4$  matrices; the SM CKM matrix is a sub block of the unitary matrix

$$V_{CKM4} = \left( \begin{array}{ccc|c} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ \hline V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{array} \right).$$

Important bounds on the possible values of  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$  come from the same  $\Delta F = 1, 2$  processes already mentioned in the vector quark scenario, see [16]. However, the strongest constraints are indeed provided by electroweak precision observables [17]; e.g. the renowned  $S$  and  $T$  parameters provide strong limits on the mixing of fourth and third generation. Still, CKM elements can be modified by quite a bit compared to the SM. Fig. 1 illustrates this for the third row elements. Especially the “small” elements  $V_{ts}$  and  $V_{td}$  can receive large relative modifications. Note that without electroweak constraints  $V_{tb}$  could be as low as 0.8 and still survive the bounds set by flavour physics alone.

Both, vector quark and extra generation, allow deviations of roughly the same large size. At first glance this result is rather surprising as one would expect loop-dominated flavour observables to be very sensitive to both, new heavy particles ( $T$  or  $t'$ ) in loops and modified  $W$  couplings due to the changed CKM element values, and thus veto any drastic modifications. However, the two types of contributions (which may be supplemented by small tree-level FCNCs for the vector quark scenario) can cancel each other to a rather large extend. In fact this does not seem unnatural as the SM CKM structure is ‘thinned’ out in order to accommodate for  $4 \times 4$  unitarity, while an additional contribution due to the new fermions has to be added. This behaviour is observed for almost all parameter sets that result in large modifications of the CKM elements [18].

### 3.2 A pinch of Warped Extra Dimensions

The rich flavour physics phenomenology of RS type models [19] has been studied numerously. While the basic ideas and features were already established some time

ago, see e.g. [20], several involved phenomenological studies of flavour observables in RS models with [21] and without [22] custodial protection were performed recently.

For brevity, we refer to Gilad Perez’s talk [5] for the details of the setup. The main features affecting the CKM matrix are:

- Mixing of the SM fermions with their KK modes: The SM CKM matrix is then just a  $3 \times 3$  sub matrix of the fermion mixing matrix. Due to the localisation of the KK wave functions close to the so-called infrared brane, mainly the  $t$  and to lesser extent the  $b$  quark will be affected by mixing with KK modes.
- As the  $W$  also develops KK modes, the zero modes can receive KK mode admixtures during EWSB. This will also modify the CKM couplings.
- Finally, depending on the way one defines the CKM matrix, the direct effect of the KK modes has to be incorporated<sup>‡</sup>. E.g. if one defines the elements via the couplings of effective four-fermion operators the whole tower of  $W$  modes<sup>§</sup> has to be taken into account.

The potential size of the unitarity violation has been studied in detail in [23] for the custodially protected setup and the largest effects are of the order

$$1 - |V_{tb}|^2 - |V_{ts}|^2 - |V_{td}|^2 \approx \mathcal{O}(5\%)$$

and, as expected, stem from the top quark. Note that in this case the CKM matrix is defined via the couplings of the  $W$ ; hence, a straightforward comparison with [23] is not possible, as the minimal RS setup was investigated using the effective theory definition of CKM matrix elements.

As can be seen in Fig. 2 the CKM elements are in a sense protected by the  $Z \rightarrow \bar{b}b$  vertex in the minimal setup; this indicates that  $\mathcal{O}(5\%)$  effects should be expected if a custodial symmetry is invoked.

## 4 Summary

Even though the SM values of the CKM matrix elements  $V_{td}$  and  $V_{ts}$  cannot be extracted directly from experiment, unitarity of the CKM matrix alone is enough to determine the absolute values with good accuracy. An overconstrained fit to the multitude of flavour observables, while sensitive to effects of new physics, does not only provide a powerful self-consistency check of the unitarity condition but also allows the determination of  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$  with very high precision.

---

<sup>‡</sup>If the  $W$  can go on-shell, like in single- $t$  production, this effect will be strongly suppressed.

<sup>§</sup>Note that the measurement of  $G_F$  via  $\mu$  lifetime will not yield the coupling of the SM  $W$  (the zero mode), but the effective sum over all modes.

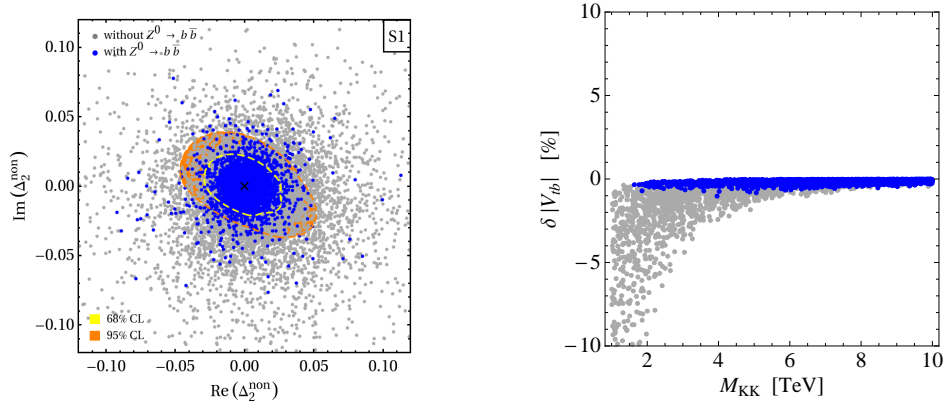


Figure 2: Unitarity violation in minimal RS model. The left plot shows the quantity  $\Delta_2 = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*$ ; the right plot shows values for  $(V_{tb}^{RS} - V_{tb}^{SM})/V_{tb}^{SM}$ . Grey points do not satisfy limits from  $Z \rightarrow b\bar{b}$ . Plots taken from [22].

However, extensions of the standard model with a non-unitary mixing matrix for the SM quarks are numerous. To provide an idea to what magnitude the  $V_{tx}$  elements can differ from their SM values we briefly discussed three models with a non-unitary 'Standard Model CKM matrix': vector-like quarks, Fourth Generation models and Warped Extra Dimensions.

Each model is capable to survive the bounds set by the various experiments and still allows for sizable modification of the CKM elements — especially the 'small' elements  $V_{ts}$  and  $V_{td}$  can deviate by  $\mathcal{O}(50\%)$ . Still, if the current experimental central value for  $V_{tb}$ , 0.88, would be strengthened by LHC and if theoretical uncertainties could be reduced accordingly all three models would be hard pressed to accommodate for this.

## ACKNOWLEDGEMENTS

I would like to thank Ulrich Haisch for illuminating discussions on the flavour structure of RS models and for providing the nice plots. Furthermore, I thank the organisers of CKM2010 for the excellent work. This work was supported in part by the DFG SFB/TR 9 "Computergestützte Theoretische Teilchenphysik".

## References

- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973). N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
- [2] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **100**, 192003 (2008).
- [3] T. E. W. Group [CDF and D0 Collaboration], arXiv:0908.2171 [hep-ex].

- [4] T. Aaltonen *et al.* [CDF Collaboration], arXiv:1004.1181 [hep-ex].
- [5] W. Wagner, these proceedings. G. Perez, these proceedings.
- [6] G. Aad *et al.* [The ATLAS Collaboration], arXiv:0901.0512 [hep-ex].
- [7] G. L. Bayatian *et al.* [CMS Collaboration], J. Phys. G **34**, 995 (2007).
- [8] T. Han, Int. J. Mod. Phys. A **23**, 4107 (2008) [arXiv:0804.3178 [hep-ph]].
- [9] <http://www.utfit.org/UTfit/Results> (11/08/10)
- [10] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41**, 1 (2005). M. Bona *et al.* [UTfit Collaboration], JHEP **0507**, 028 (2005).
- [11] C. S. Kim and H. Yamamoto, arXiv:hep-ph/0004055.
- [12] C. S. Kim and A. S. Dighe, Int. J. Mod. Phys. E **16**, 1445 (2007).
- [13] F. J. Botella, G. C. Branco and M. Nebot, Phys. Rev. D **79**, 096009 (2009).
- [14] J. Alwall *et al.*, Eur. Phys. J. C **49**, 791 (2007).
- [15] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck and C. Promberger, JHEP **1009**, 104 (2010). H. Lacker and A. Menzel, JHEP **1007**, 006 (2010).
- [16] A. Soni *et al.* Phys. Rev. D **82**, 033009 (2010). A. J. Buras *et al.* JHEP **1009**, 106 (2010). A. J. Buras *et al.* JHEP **1007**, 094 (2010).
- [17] J. Erler and P. Langacker, Phys. Rev. Lett. **105**, 031801 (2010). M. S. Chanowitz, Phys. Rev. D **79**, 113008 (2009). M. S. Chanowitz, Phys. Rev. D , 035018 (2010). O. Eberhardt, A. Lenz and J. Rohrwild, arXiv:1005.3505 [hep-ph].
- [18] M. Bobrowski *et al.* Phys. Rev. D **79**, 113006 (2009).
- [19] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [20] K. Agashe *et al.* Phys. Rev. D **71**, 016002 (2005); S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001); S. J. Huber, Nucl. Phys. B **666**, 269 (2003)
- [21] M. E. Albrecht *et al.* JHEP **0909**, 064 (2009); M. Blanke *et al.* JHEP **0903**, 108 (2009); M. Blanke *et al.* JHEP **0903**, 001 (2009); S. Casagrande, F. Goertz, U. Haisch, M. Neubert and T. Pfoh, JHEP **1009**, 014 (2010)
- [22] M. Bauer, S. Casagrande, U. Haisch and M. Neubert, JHEP **1009**, 017 (2010); S. Casagrande *et al.* JHEP **0810**, 094 (2008);
- [23] A. J. Buras, B. Duling and S. Gori, JHEP **0909**, 076 (2009).